Heteroscedastic Uncertainty Estimation Framework for Unsupervised Registration

Project page Paper

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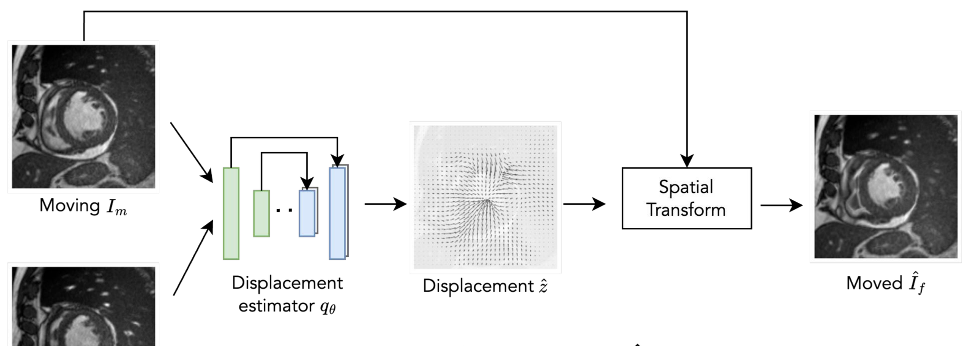
Summary

We propose an unsupervised image registration framework that extends the commonly used homoscedastic noise assumption to heteroscedastic.

Introduction

Homoscedastic means uniform; heteroscedastic refers to non-uniform.



$$\mathcal{L} = \underbrace{\frac{1}{|\Omega|}\sum_{\Omega}[I_f - \hat{I}_f]^2}_{\mathcal{L}_{\text{data}}} + \lambda\|\nabla\hat{z}\|^2$$

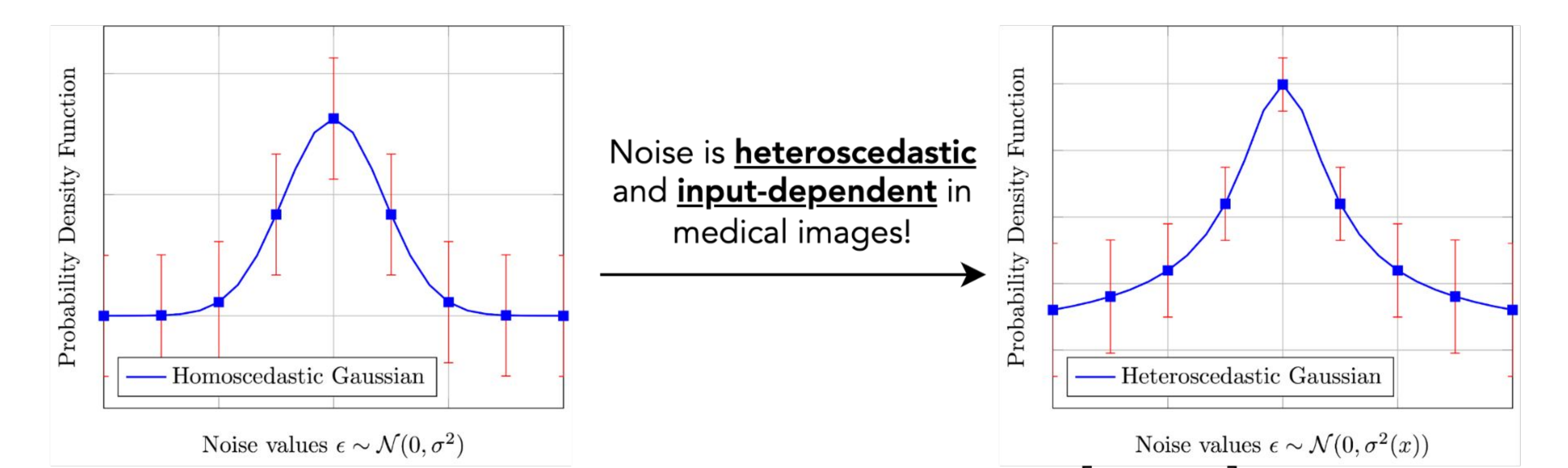
- Unsupervised registration aims to align two images without any labels.
- Existing methods commonly use a intensity-based objective, such as mean squared error.

Motivation

Existing image registration approaches often assume homoscedastic noise, whereas real-world medical images exhibit heteroscedastic noise.

Existing approaches: Simplified **homoscedastic** noise assumption

Ours approach: Adaptive exponentiated SNR weighting



$$\mathcal{L} = [I_f - \hat{I}_f]_2^2 + \lambda\|\nabla\hat{z}\|^2$$

$$\mathcal{L} = \tau\left[\left(\frac{I_f}{\lfloor\hat{\sigma}_I\rfloor}\right)^{2\gamma}\right][I_f - \hat{I}_f]_2^2 + \lambda\|\nabla\hat{z}\|^2$$

Preliminary

Modeling heteroscedastic noise as aleatoric uncertainty

- Two types of uncertainty: aleatoric and epistemic.
- Heteroscedastic noise is a type of aleatoric uncertainty, which can be expressed as: $Y = \mu(X) + \epsilon(X) \quad \epsilon(X) = \mathcal{N}(0, \sigma^2(X))$
- The following objectives learn the input-dependent mean and variance:

$$\mathcal{L}_{\text{NLL}} = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{2\hat{\sigma}^2(x_i)}\|y_i - \hat{\mu}(x_i)\|_2^2 + \frac{1}{2}\hat{\sigma}^2(x_i)$$

$$\mathcal{L}_{\beta\text{-NLL}} = \frac{1}{N}\sum_{i=1}^{N}\lfloor\hat{\sigma}^{2\beta(x_i)}\rfloor\left(\frac{1}{2\hat{\sigma}^2(x_i)}\|y_i - \hat{\mu}(x_i)\|_2^2 + \frac{1}{2}\hat{\sigma}^2(x_i)\right)$$

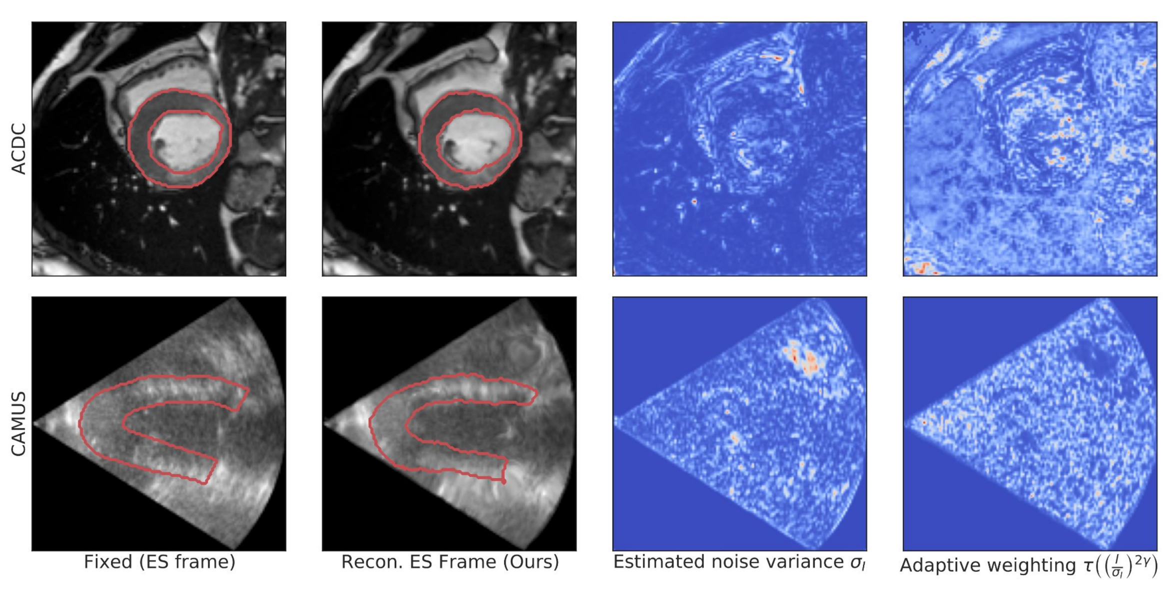
Naive integration to image registration

- Fixed is a noisy observation of the moved: $p(I_f|\hat{z}; I_m) = \mathcal{N}(\hat{I}_f; I_m, \hat{\sigma}_I^2)$
- Preliminary objective: $\mathcal{L} = \mathbb{E}_\Omega\left[\frac{1}{\hat{\sigma}_I^2}[I_f - \hat{I}_f]_2^2 + \log\hat{\sigma}_I^2 + \lambda\|\nabla\hat{z}\|^2\right]$

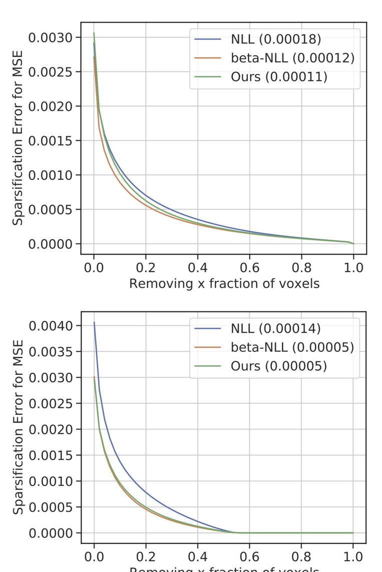
Evaluation on heteroscedastic uncertainty

Sparsification error: remove one pixel at a time from largest to smallest uncertainty magnitudes, measured the MSE of the remaining pixels.

Qualitative visualization

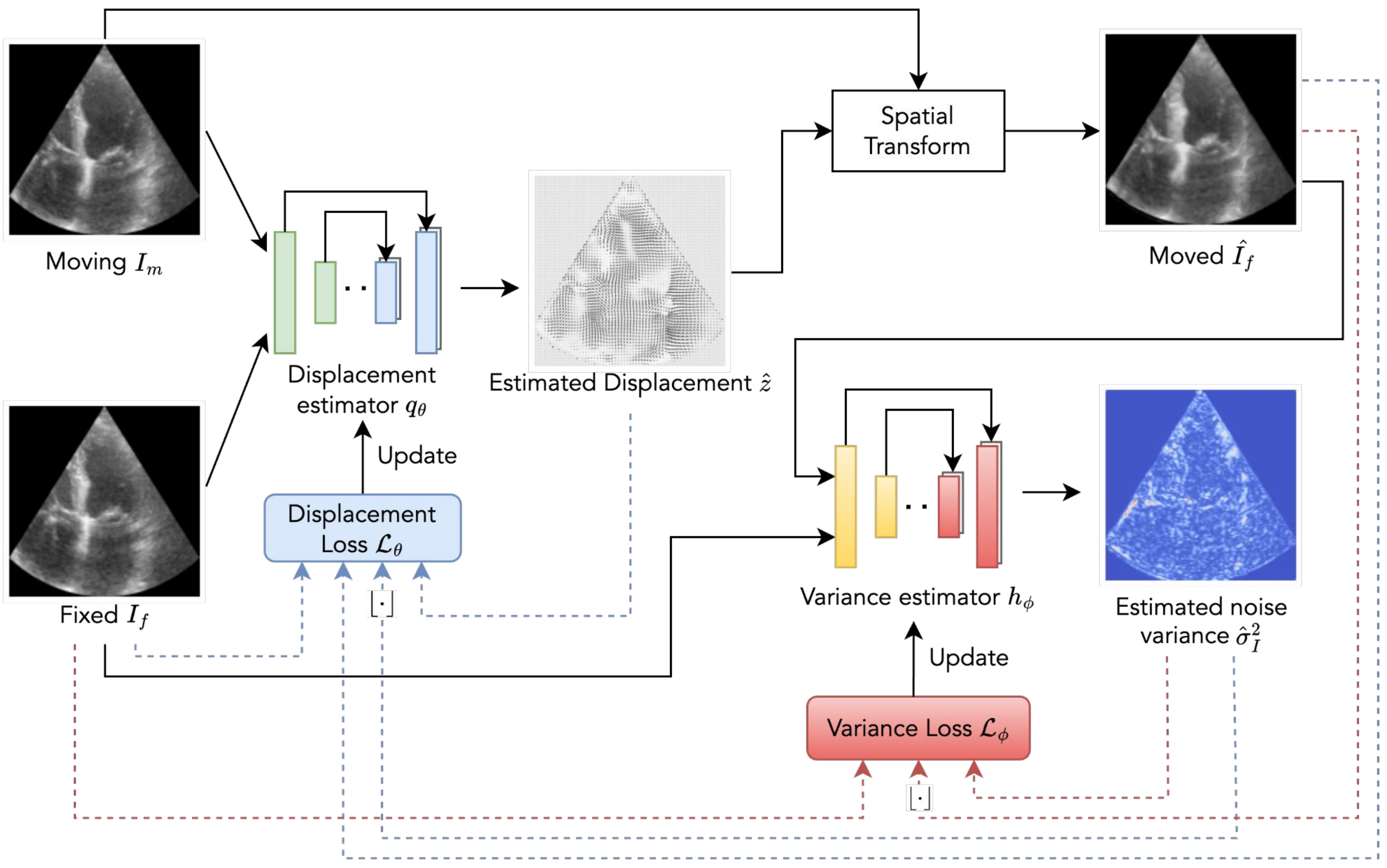


Sparsification error



Our estimated uncertainty is visually sensible and quantitatively supported by sparsification error metrics.

Methods



Displacement estimator loss

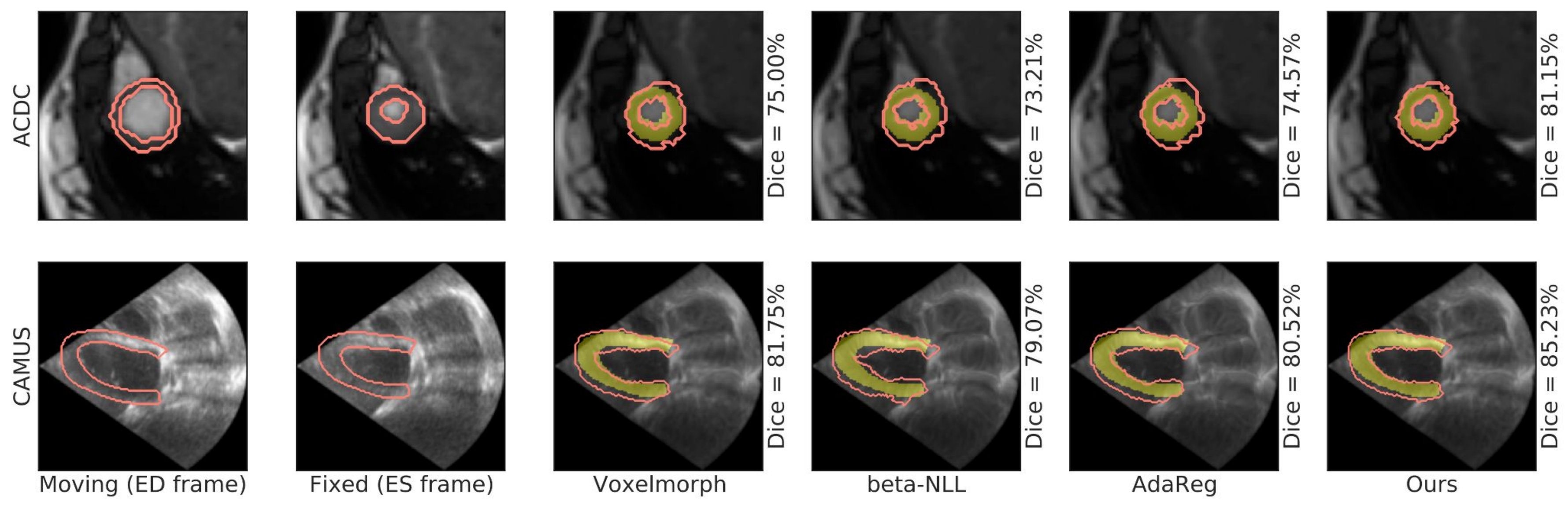
$$\mathcal{L}_\theta = \mathbb{E}_\Omega\left[\mathcal{T}\left[\left(\frac{I_f}{\lfloor\hat{\sigma}_I\rfloor}\right)^{2\gamma}\right][I_f - \hat{I}_f]_2^2 + \lambda\|\nabla\hat{z}\|^2\right]$$

Variance estimator loss

$$\mathcal{L}_\phi = \mathbb{E}_\Omega\left[\lfloor\hat{\sigma}_I^{2\beta}\rfloor\left(\frac{1}{\hat{\sigma}_I^2}[I_f - \lfloor\hat{I}_f\rfloor]_2^2 + \log\hat{\sigma}_I^2\right)\right]$$

- A relative weighting instead of absolute $p(x) \sim \frac{1}{\hat{\sigma}_I(x)} \to p(x) \sim \frac{I_f(x)}{\hat{\sigma}_I(x)}$
- $\boldsymbol{\gamma}$ controls confidence of current noise variance estimate.

Registration accuracy



		ACDC			CAMUS		
		DSC ↑	HD ↓	ASD ↓	DSC ↑	HD ↓	ASD ↓
	Undeformed	47.98	7.91	2.32	66.77	10.87	2.61
	Elastix [3]	77.26	4.95	1.28	80.18	10.02	1.81
CNN	vxm (NCC)	78.55	4.94	1.29	77.01	10.23	1.89
CNN	vxm (MI)	78.04	5.25	1.35	78.18	9.83	1.99
CNN	vxm (MSE) †	80.20	4.64	1.24	81.76	8.93	1.70
CNN	NLL	76.49	5.46	1.45	75.24	11.05	2.20
CNN	β-NLL	78.74	5.07	1.33	79.75	9.39	1.93
CNN	AdaFrame	66.38	5.80	1.67	77.88	10.54	1.93
CNN	AdaReg	78.75	5.13	1.33	79.31	9.78	1.88
CNN	Ours	**80.73**	**4.57**	**1.21**	**81.96**	**8.80**	**1.66**
Transformer	tsm (NCC)	73.77	6.64	1.12	73.03	11.87	**1.70**
Transformer	tsm (MI)	73.57	6.57	**1.11**	74.83	11.94	1.83
Transformer	tsm (MSE) †	76.94	5.51	1.30	79.24	10.30	1.79
Transformer	NLL	73.12	7.22	1.27	75.08	11.60	1.79
Transformer	β-NLL	75.74	6.12	1.29	77.39	10.99	1.86
Transformer	AdaFrame	67.95	5.72	1.59	78.06	9.86	1.91
Transformer	AdaReg	76.22	5.68	1.29	78.12	10.62	1.84
Transformer	Ours	**78.12**	**5.04**	1.26	**80.38**	**9.86**	1.72

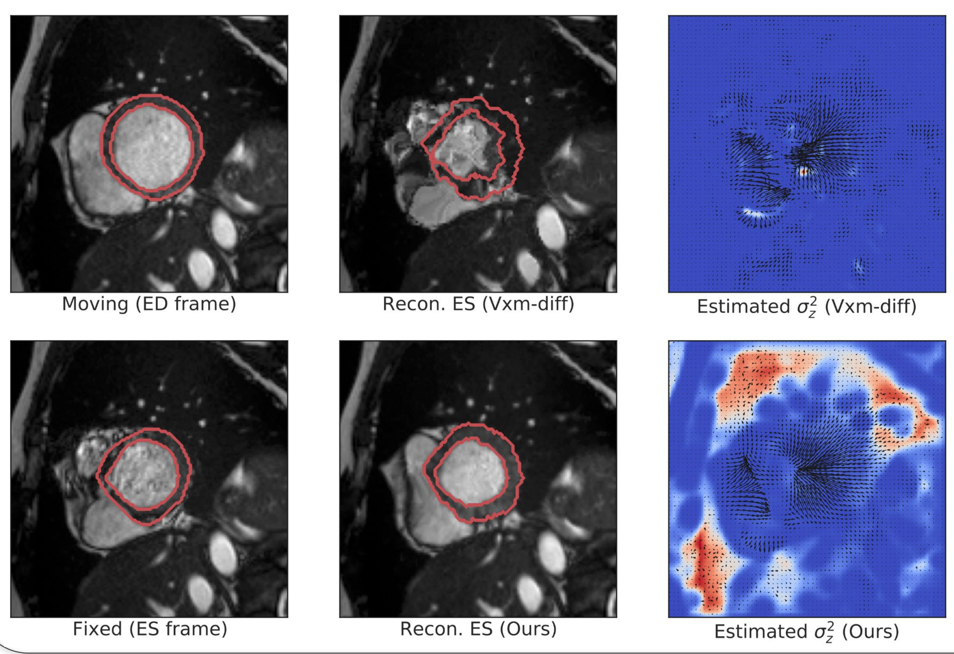
Effect of $\boldsymbol{\gamma}$

	ACDC			CAMUS		
	DSC ↑	HD ↓	ASD ↓	DSC ↑	HD ↓	ASD ↓
Ours ($\gamma = 0.25$)	79.74	4.74	1.26	**82.07**	8.53	**1.65**
Ours ($\gamma = 0.5$)	**80.73**	**4.57**	**1.21**	81.96	8.80	1.66
Ours ($\gamma = 0.75$)	80.00	4.69	1.24	81.82	**8.45**	1.66
Ours ($\gamma = 1$)	79.78	4.71	1.25	81.31	9.08	1.69

- We evaluate the proposed framework by registering end-diastole (ED) frame to end-systole frame (ES).
- Our proposed approach consistently outperforms baselines in various architectures and datasets

Incorporating displacement uncertainty

Displacement loss: $\mathcal{L}_\theta = \mathbb{E}_\Omega\left[\mathcal{T}\left[\left(\frac{I_f}{\lfloor\hat{\sigma}\rfloor_I}\right)^{2\gamma}\right][I_f - \hat{I}_f]_2^2 + \alpha\left(\hat{\sigma}_z^2 - \log\hat{\sigma}_z^2\right) + \lambda\|\nabla\hat{z}\|^2\right]$



Quantitative evaluation

	Uncertainty		ACDC			CAMUS		
	σ_z^2	σ_I^2	DSC ↑	HD ↓	ASD ↓	DSC ↑	HD ↓	ASD ↓
Vxm	✗	✗	80.20	4.64	1.24	81.76	8.93	1.70
Vxm-diff	✓	✗	76.19	5.75	**1.19**	76.74	10.76	1.88
Ours	✓	✗	79.80	4.74	1.22	81.47	8.67	1.69
Ours	✗	✓	**80.73**	**4.57**	1.21	**81.96**	8.80	1.66
Ours	✓	✓	79.87	4.62	1.20	81.91	**8.54**	**1.65**

Our framework is versatile to incorporate displacement uncertainty.

Paper: https://arxiv.org/abs/2312.00836 Code: https://github.com/Voldemort108X/hetero_uncertainty Funding: NIH/NHLBI grant R01HL121226.